A Universe in Heidelberg

Based on:

Nature 611, 260-264

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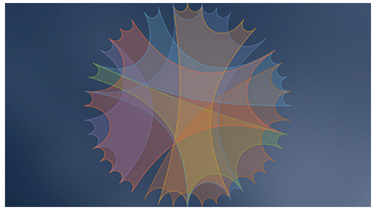

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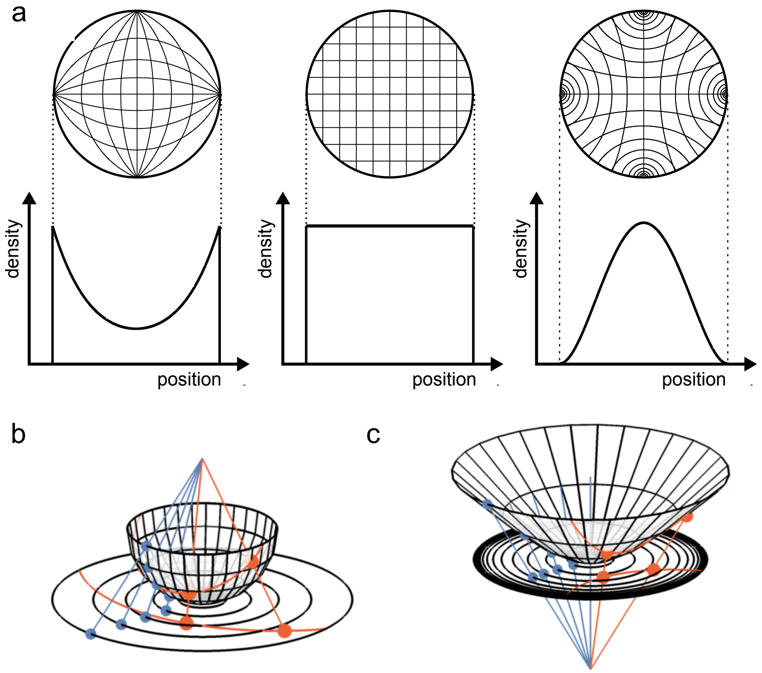


The Team

- Experiment:


Celia Viermann


Marius Sparn


Nikolas Liebster


Maurus Hans

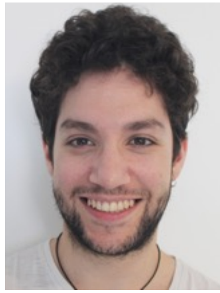

Elinor Kath


Helmut Strobel


Markus K. Oberthaler

- Theory:


Mireia Tolosa-Simeón


Álvaro Parra-López

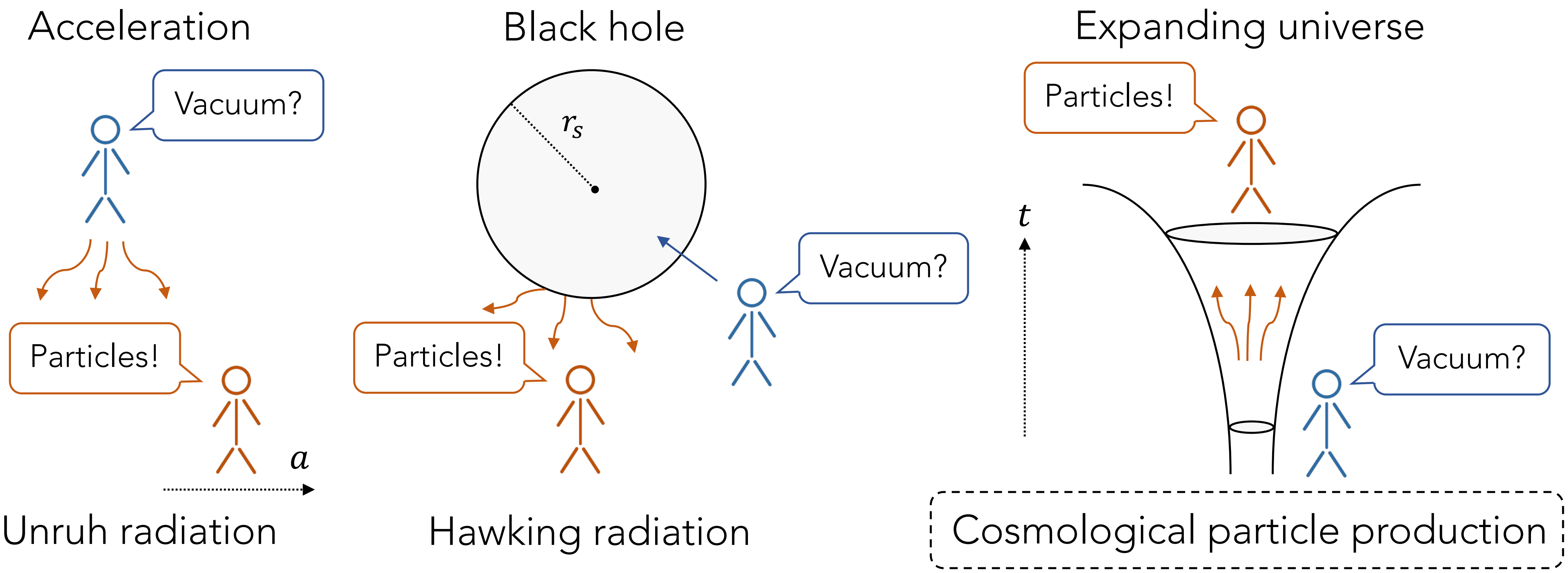

Natalia Sánchez-Kuntz


Tobi Haas


Stefan Flörchinger

Big picture - I

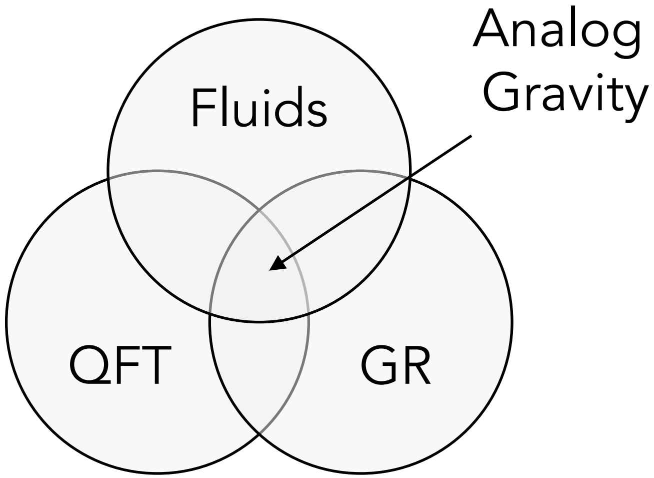
- Quantum fields on curved spacetime Birrel et al. '82, Mukhanov et al. '07



→ Notions of vacua and particles become more involved

Big picture - II

- Major challenges
  - Phenomena hard to observe in the sky
  - Test objects dictated by nature
- Key idea Unruh '81, '89, Visser et al. '02, Novello et al. '02



→ Cosmological problems ⟷ Fluids

→ Try to observe analog cosmological particle production Jain et al. '07, Eckel et al. '18

Outline

Theory

- Mapping BEC $\leftrightarrow$ Universe
- Cosmological particle production

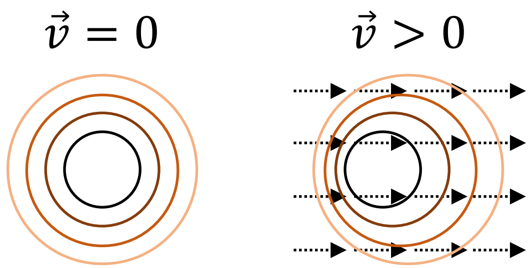
Experiment

- Phonon propagation
- Density fluctuations

A simple analogy - I

Mapping BEC ↔ Universe | Phonon propagation | Cosmological particle production | Density fluctuations

- Sound waves in a fluid[Novello et al. '02], which is



- irrotational $\vec{\nabla}\times\vec{v} = 0 \rightarrow \vec{v} = \vec{\nabla}\phi$
- inviscid $\mu = 0$
- barotropic $p = p(\rho)$

- Fluid described by

Continuity equation $\frac{\partial \rho}{\partial t} + \vec{\nabla}(\rho\vec{v}) = 0,$ Euler equation $\rho\left(\frac{\partial \vec{v}}{\partial t} + (\vec{v}\cdot\vec{\nabla})\vec{v}\right) = -\nabla p$

- Sound waves = linear fluctuations around background

$$\rho \rightarrow \rho_0 + \rho_1, \qquad p \rightarrow p_0 + p_1, \qquad \phi \rightarrow \phi_0 + \phi_1$$

A simple analogy - II

- E.o.m. of sound waves

$$\frac{1}{\sqrt{g}}\frac{\partial}{\partial x^\mu}\left(\sqrt{g}\, g^{\mu\nu}\frac{\partial}{\partial x^\nu}\,\phi_1\right) = 0$$

with

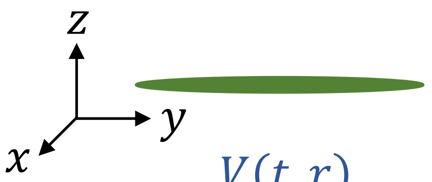
  - acoustic metric $g_{\mu\nu} \propto \begin{pmatrix} -(c^2 - v_0^2) & -\vec{v}_0 \\ -\vec{v}_0 & 1 \end{pmatrix}$
  - speed of sound $c^2 = \frac{\partial p}{\partial \rho}$

→ Sound waves ↔ Scalar field in curved spacetime

→ Control geometry by shaping the fluid flow

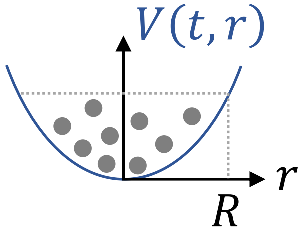
Two-dimensional BEC

- Experimental setup
  - Pancake trap



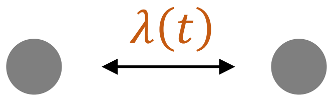
→ 2D geometry

  - Isotropic trap



→ $V(t,r) = \frac{m}{2}\omega^2(t) f(r)$

  - Interactions



→ $\lambda(t) \propto a_s(t)$

- Effective action of Bose-Einstein condensate (BEC) Gross '61, Pitaevskii '61

$$\Gamma[\Phi] = \int dt\, d^2r \left\{ \hbar\Phi^* \left[i\frac{\partial}{\partial t} - V(t,r) \right] \Phi - \frac{\hbar^2}{2m} (\vec{\nabla}\Phi^*)(\vec{\nabla}\Phi) - \frac{\lambda(t)}{2} (\Phi^*\Phi)^2 \right\}$$

Dynamics of Phonons

- Split into background and fluctuations

$$\Phi(t,\vec{r}) \rightarrow \phi_0(t,\vec{r}) + \frac{1}{2}[\phi_1(t,\vec{r}) + i\phi_2(t,\vec{r})]$$

- Effective action of phonons ($\phi = \phi_2/\sqrt{2m}$)

$$\Gamma[\phi] = -\frac{\hbar^2}{2}\int dt\, d^2r\, \sqrt{g}\, g^{\mu\nu}\, \frac{\partial}{\partial x^\mu}\, \phi\, \frac{\partial}{\partial x^\nu}\, \phi$$

in acoustic approx. $\omega(k) = c\, k$ and for $\vec{v}_0 = \text{const.}$

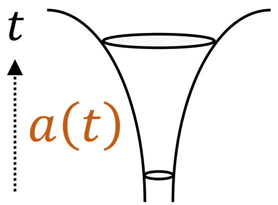
→ Phonons ↔ Relativistic, free, massless, scalar field in curved spacetime

Acoustic metric vs. FLRW metrics

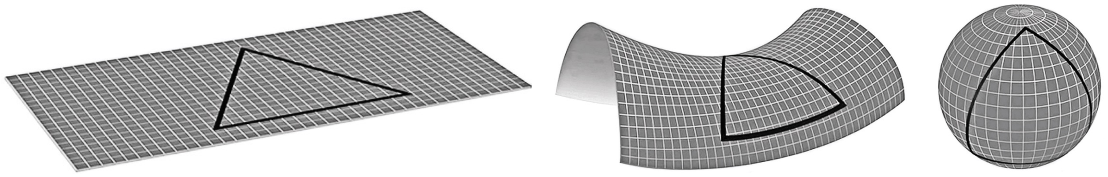
- Spacetime geometry determined by
  - acoustic metric $g_{\mu\nu} = \frac{1}{c^2}\begin{pmatrix} -(c^2 - v_0^2) & -\vec{v}_0 \\ -\vec{v}_0 & 1 \end{pmatrix}$
  - speed of sound $c^2(t,\vec{r}) = \frac{\lambda(t)\, n_0(t,\vec{r})}{m}$ ⟵ time- and space-dependent!
- Friedmann-Lemaître-Robertson-Walker (FLRW) metrics Weinberg '08

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -c^2dt^2 + a^2(t)\left(\frac{du^2}{1-\kappa u^2} + u^2 d\Omega^2\right)$$

Scale factor



Spatial curvature



$\kappa = 0$ $\kappa < 0$ $\kappa > 0$

Engineering expansion

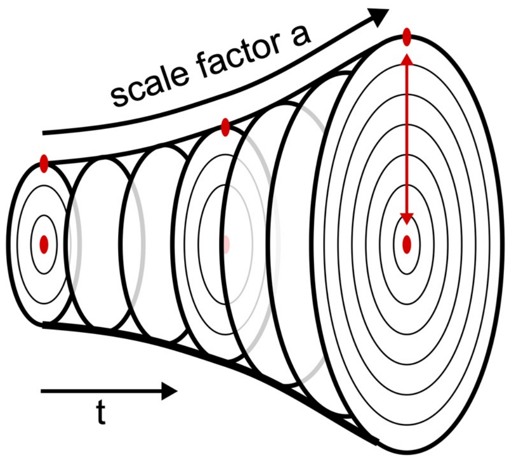
- For scale factor $a(t)$ Jain et al. '07
  - Just define

$$a^2(t) = \frac{m}{\bar{n}_0}\frac{1}{\lambda(t)} \propto \frac{1}{\bar{c}^2(t)}$$

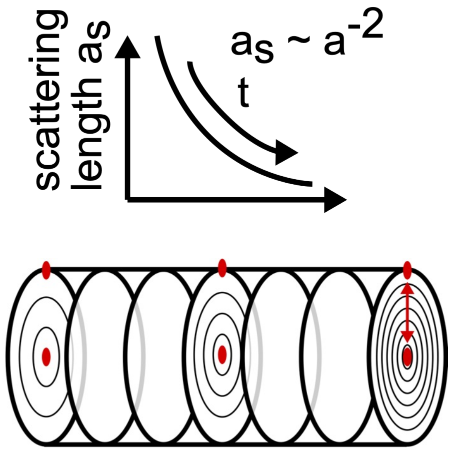
  - and note the equivalence

    expanding space ↔

    decreasing causal speed



↔



→ Coupling $\lambda(t)$ ↔ Scale factor $a(t)$

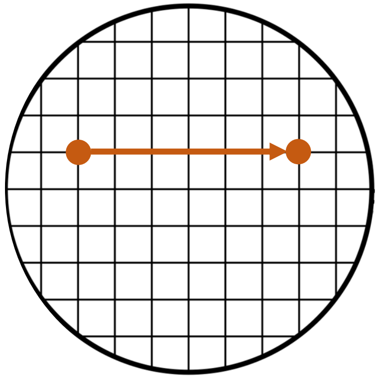
Engineering curvature - I

- For flat universe $\kappa = 0$

Density profile

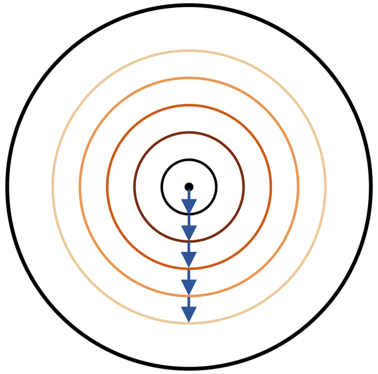
$n_0(r) = \bar{n}_0$

Geometry



Geodesics:
straight

Phonons

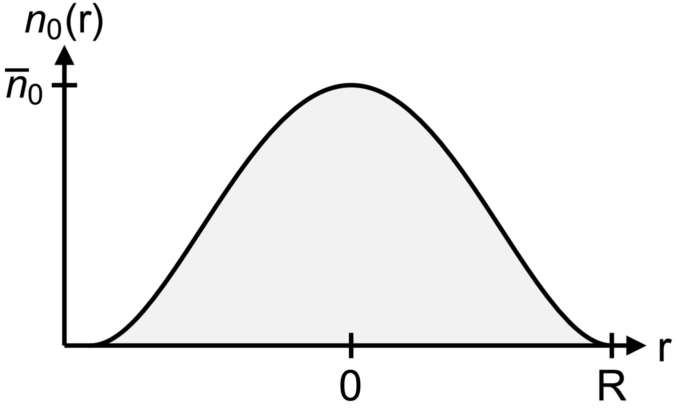


Radial distance:
constant

Engineering curvature - II

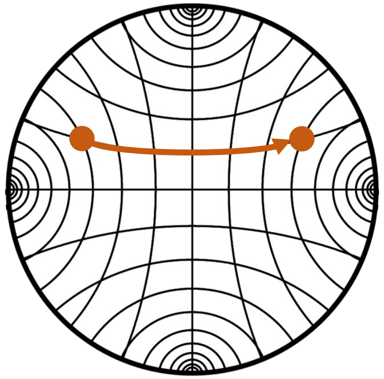
- For hyperbolic universe $\kappa < 0$

Density profile



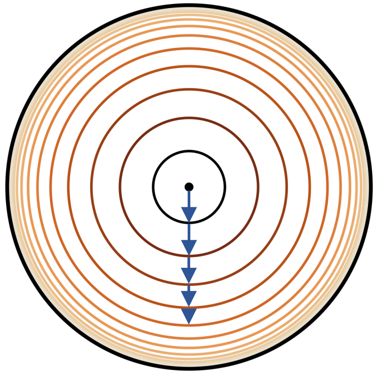
$$n_0(r) = \bar{n}_0 \left(1 - \frac{r^2}{R^2}\right)^2$$

Geometry



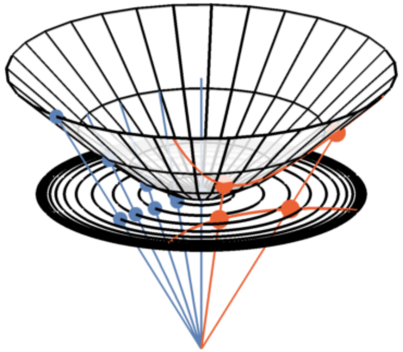
Geodesics:
arcs + dia.

Phonons



Radial distance:
decreases

Projection



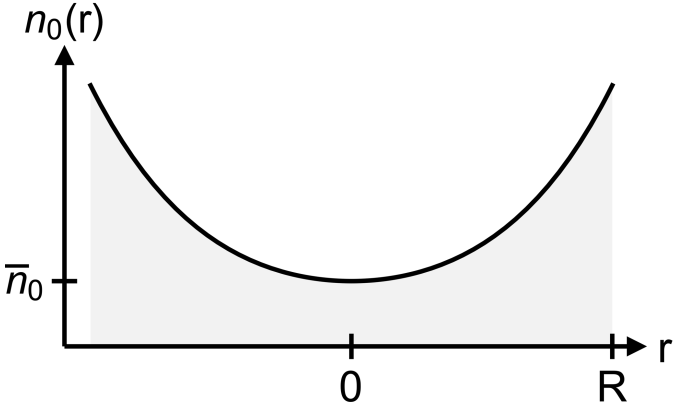
Poincaré disc

→ Density profile $n_0(r)$ ↔ Spatial curvature $\kappa = -4/R^2$

Engineering curvature - III

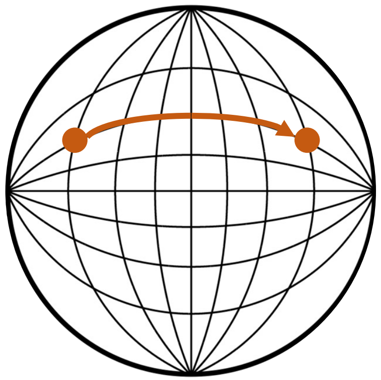
- For spherical universe $\kappa > 0$

Density profile



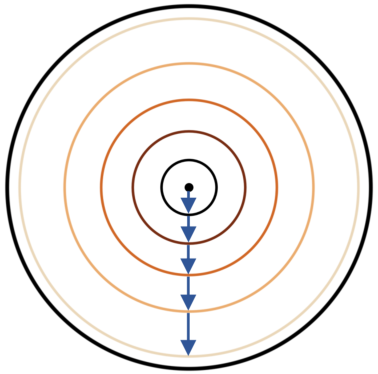
$$n_0(r) = \bar{n}_0 \left(1 + \frac{r^2}{R^2}\right)^2$$

Geometry



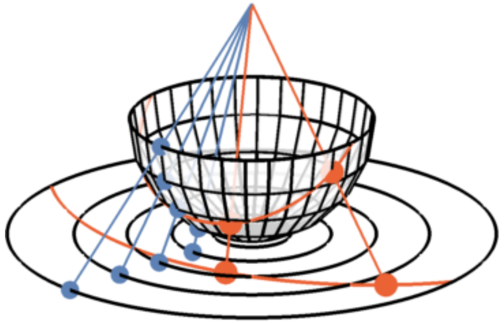
Geodesics:
arcs + dia.

Phonons



Radial distance:
increases

Projection

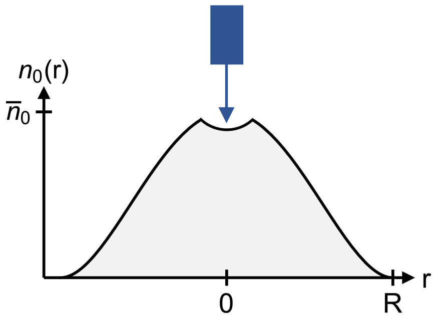


Stereographic disc

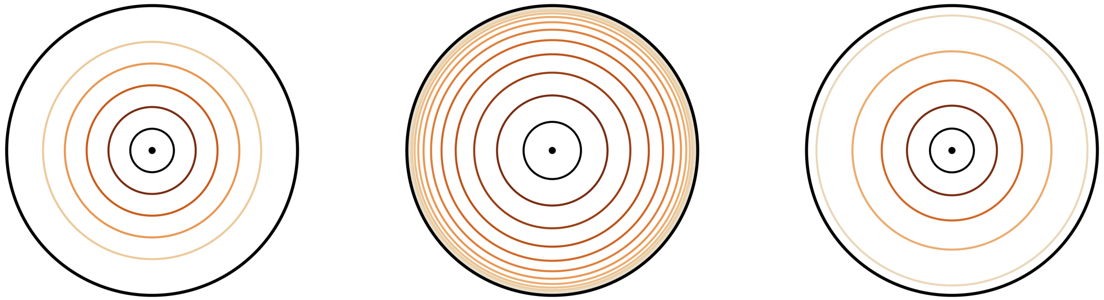
→ Density profile $n_0(r)$ ↔ Spatial curvature $\kappa = +4/R^2$

Imprinting phononic waves

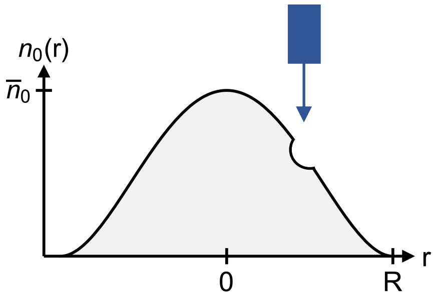
- Perturb condensate with a laser
  - In the center



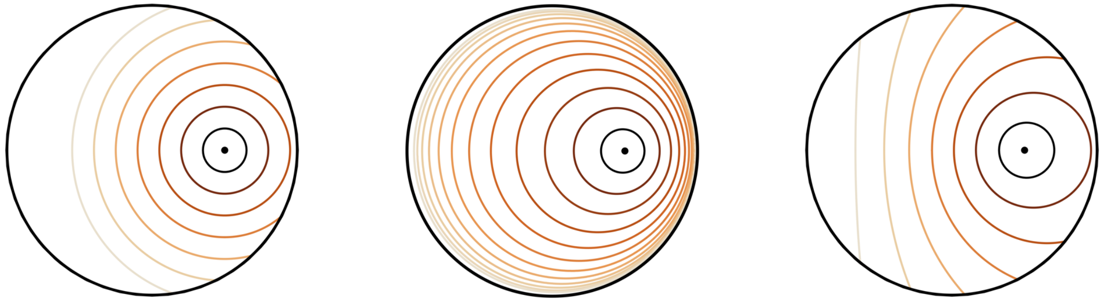
$\kappa = 0$ $\kappa < 0$ $\kappa > 0$



  - Somewhere else

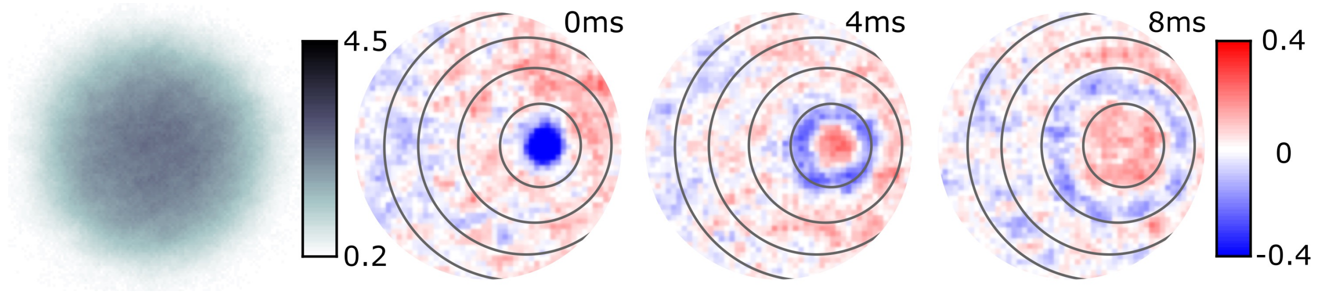


$\kappa = 0$ $\kappa < 0$ $\kappa > 0$

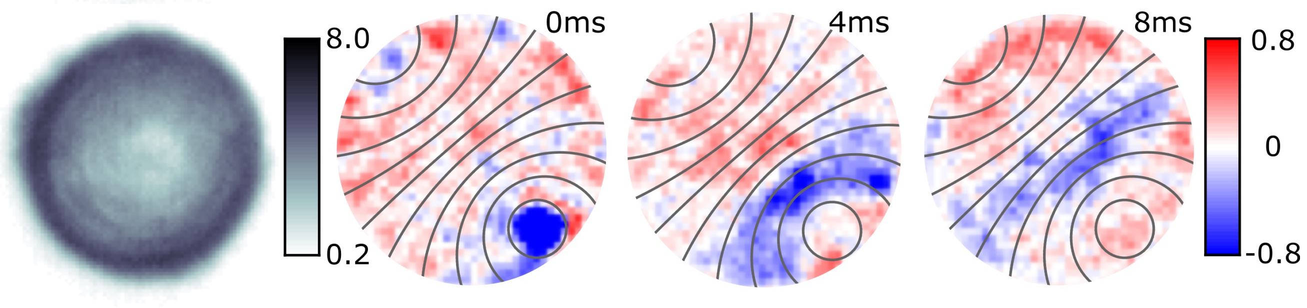


Configurability of spatial curvature

- Hyperbolic geometry ($\kappa < 0$)

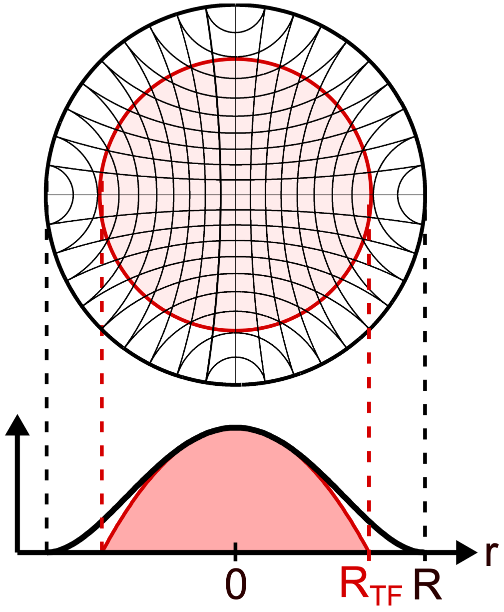


- Spherical geometry ($\kappa > 0$)



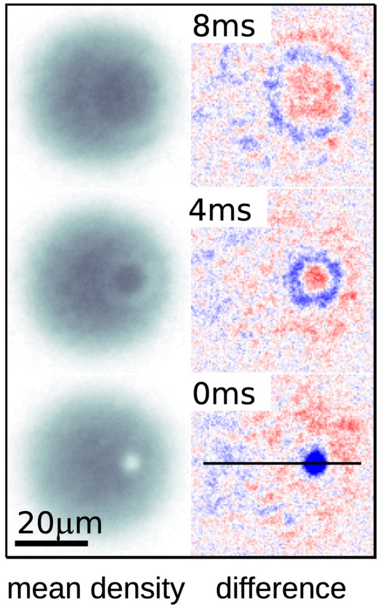
Phonon trajectories

- Hyperbolic geometry ($\kappa < 0$)

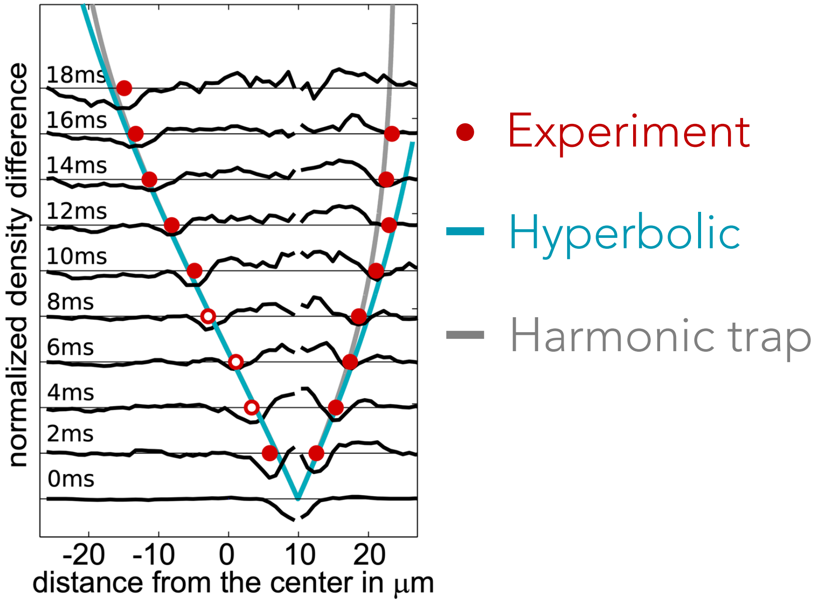


Harmonic trap

$R = \sqrt{2}\, R_{TF}$



Phonon propagation



Quantitative comparison

Quantization of phonon field

- E.o.m. of phonon field ϕ with FLRW metric $g_{\mu\nu}$

$$\ddot{\phi} + 2\frac{\dot{a}(t)}{a(t)}\dot{\phi} - \frac{1}{a^2(t)}\Delta\phi = 0$$

- Promote phonon field ϕ to a quantum operator $\hat{\phi}$

$$\hat{\phi}(t,u,\varphi) = \int_{k,m} \left[\hat{a}_{km}\,\mathcal{H}_{km}(u,\varphi)\,v_k(t) + \hat{a}^\dagger_{km}\,\mathcal{H}^*_{km}(u,\varphi)\,v^*_k(t)\right]$$

Annihilation op.

$$\Delta\mathcal{H}_{km}(u,\varphi) = h(k)\mathcal{H}_{km}(u,\varphi)$$

Mode functions

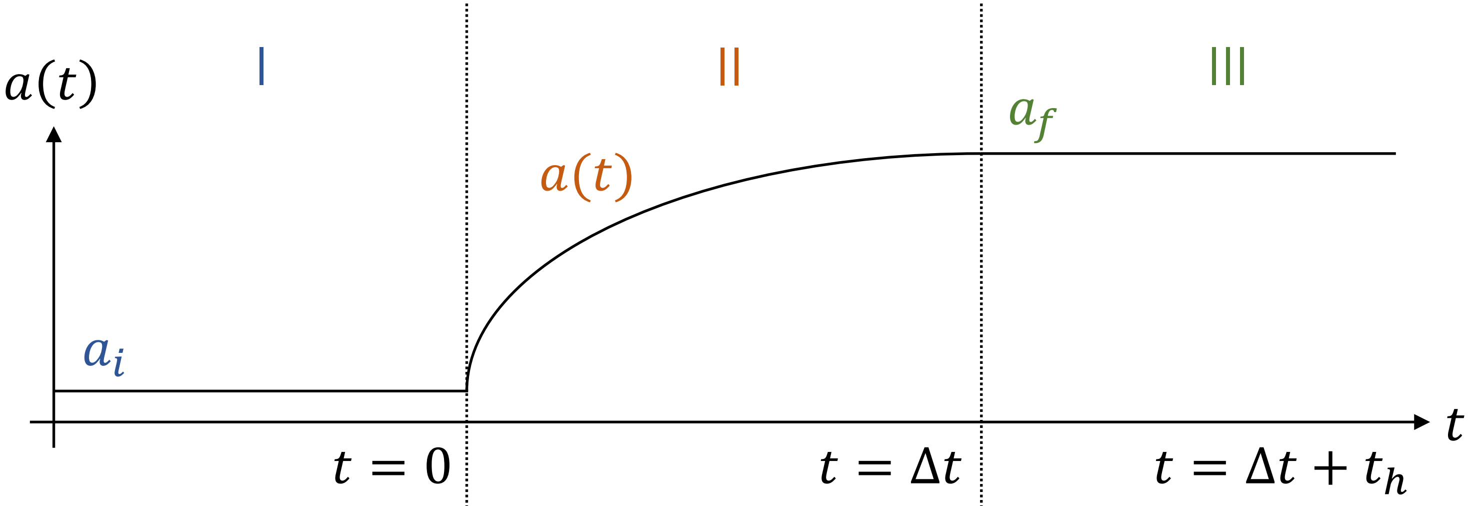
- Obtain mode equation

$$\ddot{v}_k(t) + 2\frac{\dot{a}(t)}{a(t)}\dot{v}_k(t) - \frac{h(k)}{a^2(t)}v_k(t) = 0$$

Effects of expansion - I

- Controlled expansion Jain et al. '07

  - Regions



  - Mode functions: I: $v_k(t) \propto e^{-i\omega_k^I t}$ | II: depends on $a(t)$ | III: $u_k(t) \propto e^{-i\omega_k^{III} t}$
  - Particles/Vacua: I: $\hat{a}_{km}|\Omega\rangle = 0$ | II: ambiguous | III: $\hat{b}_{km}|\Psi\rangle = 0$

Effects of expansion - II

- Initial vacuum $|\Omega\rangle$ is not empty after the expansion

$$\hat{b}_{km}|\Omega\rangle \neq 0$$

→ Particles get produced

- Relate quantities before and after expansion: Bogoliubov transformation
  - Mode functions $$v_k = \alpha_k^* u_k - \beta_k u_k^*$$
  - Annihilation operator $$\hat{b}_{km} = \alpha_k^* \hat{a}_{km} - \beta_k^* (-1)^m \hat{a}_{k,-m}^\dagger$$

Bogoliubov coefficients (α_k^*, β_k^*)

- Calculate α_k, β_k by solving the mode equation in all three regions

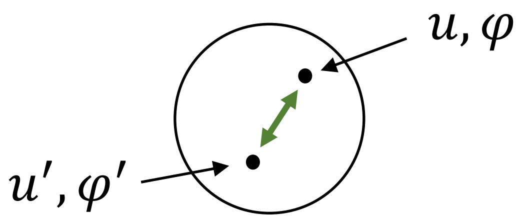
Correlation functions

- Rescaled density contrast

$$\delta_c(t,u,\varphi) = \sqrt{\frac{n_0(u)}{\bar{n}_0^3}}\,[n(t,u,\varphi) - n_0(u)]$$

with $n(t,u,\varphi) = |\Phi(t,u,\varphi)|^2$

- Correlations between two points after expansion $t \geq \Delta t$



$$\mathcal{G}_{nn}(t;u,u',\varphi,\varphi') = \langle \delta_c(t,u,\varphi)\delta_c(t,u',\varphi')\rangle$$

$$= \text{const.} \times \mathcal{G}_{\dot{\phi}\dot{\phi}}(t,L)$$

- FLRW symmetries carry over → Correlations depend only on distance L

Spectra

- Computation within FLRW paradigm gives

$$\mathcal{G}_{nn}(t, L) = \text{const.} \times \int_k \mathcal{F}(k, L) \sqrt{-h(k)}\, S_k(t)\, \tilde{f}_G(k)$$

with spectrum of fluctuations

$$S_k(t) = \frac{1}{2} + N_k + \Delta N_k(t) = \frac{1}{2} + |\beta_k|^2 + |\alpha_k \beta_k| \cos(2\omega_k t + \Theta_k)$$

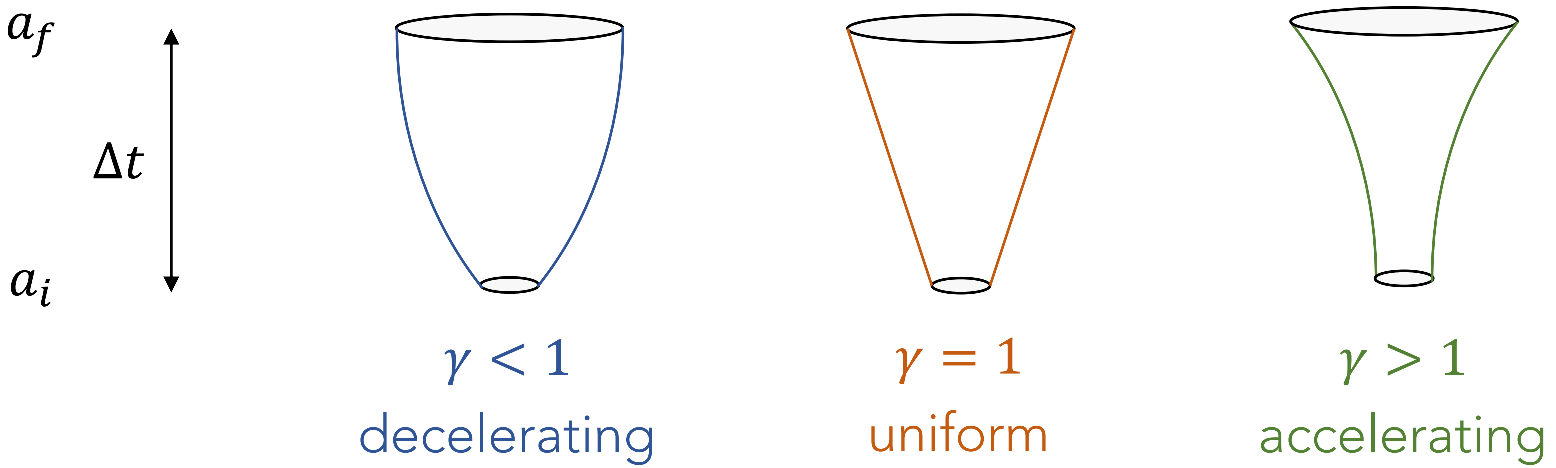
- Other things to take into account
  - Initial mode occupations N_k^{in} → Global factor $S_k(t) \times \left(1 + 2\, N_k^{in}\right)$
  - UV regularization → Convolve fields with window function $W(\vec{r})$

Expansion scenarios

- Family of power-law expansions

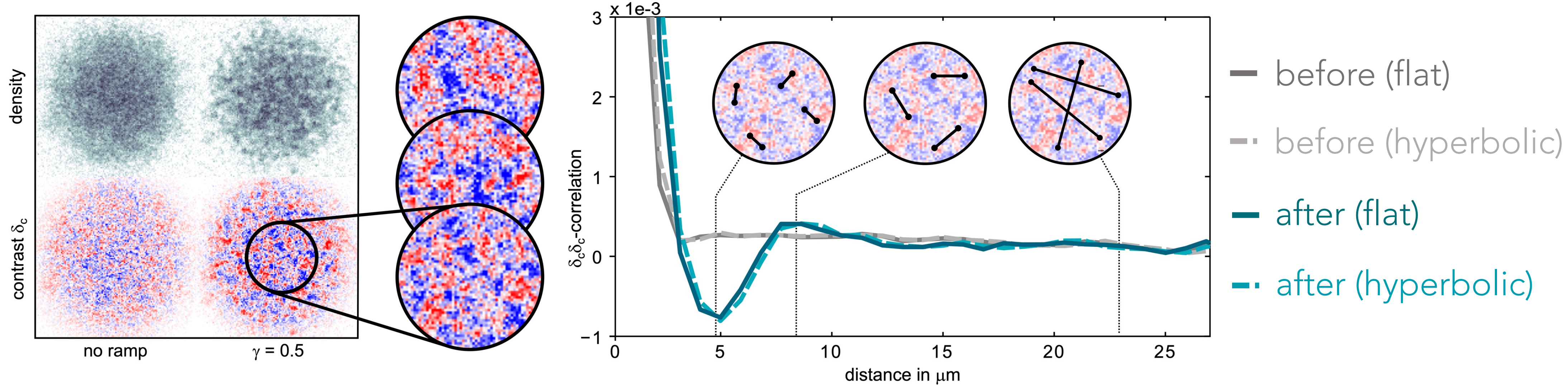
$$a(t) = \text{const.} \times |t - t_0|^\gamma$$

- Fix initial/final sizes a_i, a_f and expansion duration Δt → Three expansion classes



Extracting correlation functions

- Examples for $\gamma = 0.5$ (decelerating) and $\kappa < 0$ (harmonic trap) at $t = \Delta t = 1.5$ ms

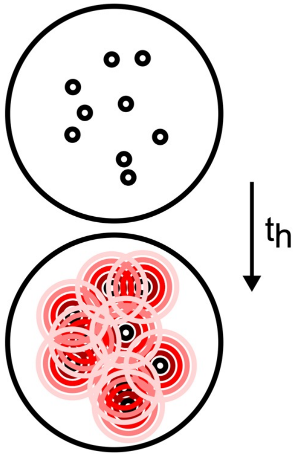


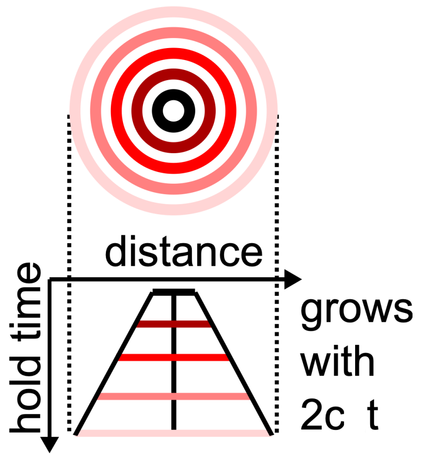
single realization	inner region	averaged over 100 realizations and all distances L

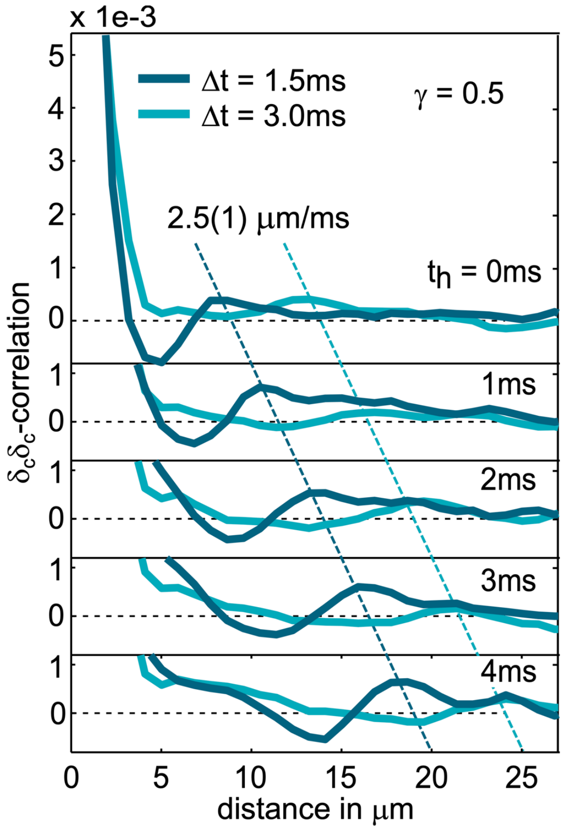
→ Clear signal and flat approximation works quite well in the center

Propagation of correlations

- Again $\gamma = 0.5$ (decelerating) and $\kappa < 0$ (harmonic trap), but now $t = \Delta t + t_h$



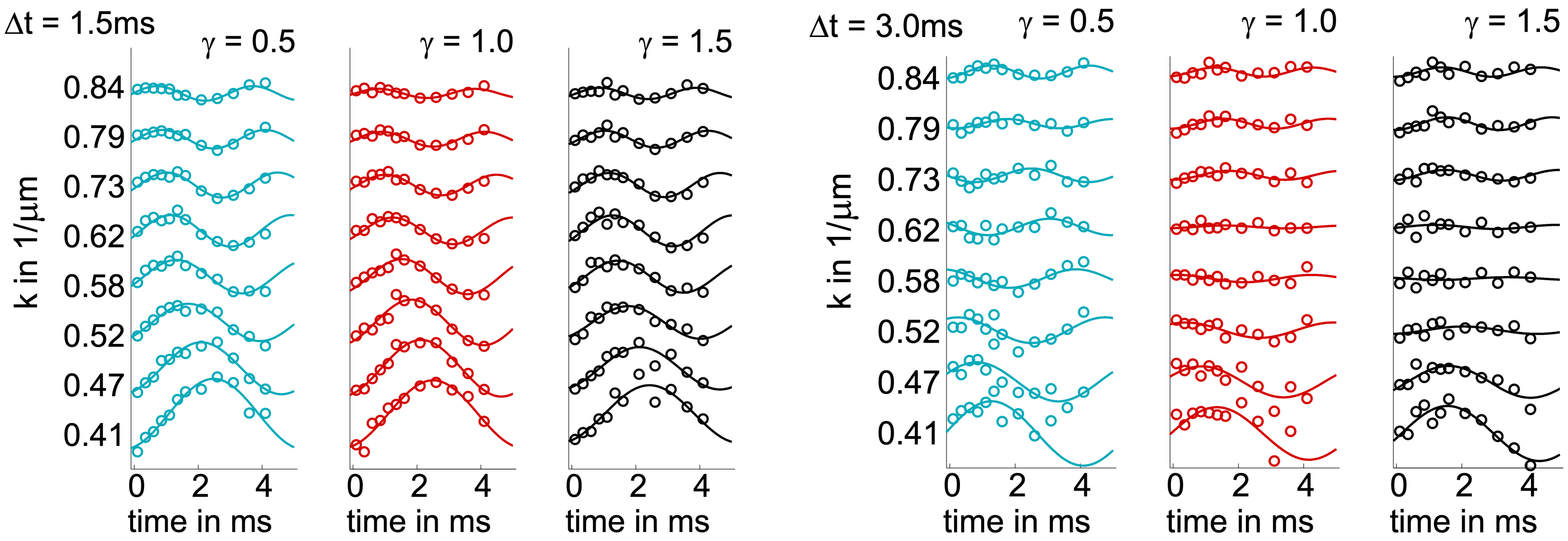




→ Correlations propagate with $2c$

Extracting spectra

- Time evolution of k-modes for all three classes and $\kappa < 0$ (harmonic trap)

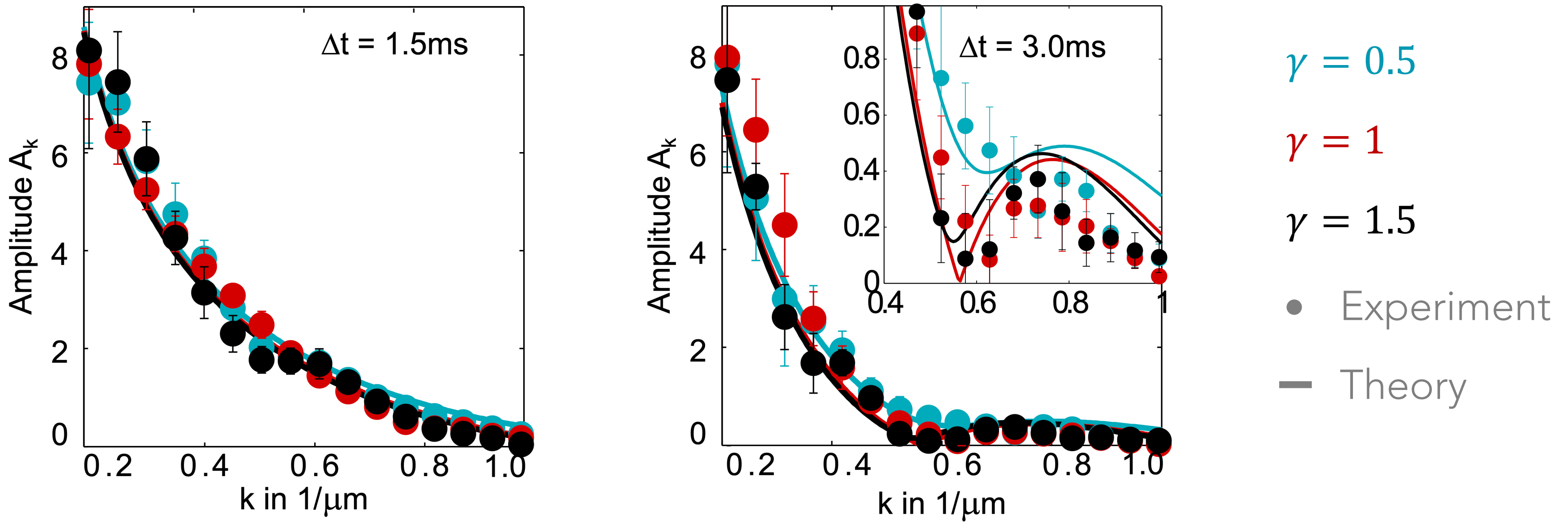


- Fit to $S_k(t) = \frac{1}{2} + |\beta_k|^2 + |\alpha_k \beta_k| \cos(2\omega_k t + \Theta_k)$

  → Extract amplitude $A_k = |\alpha_k \beta_k|$ and phase Θ_k of k-modes

Amplitudes

- For $\kappa < 0$ (harmonic trap)

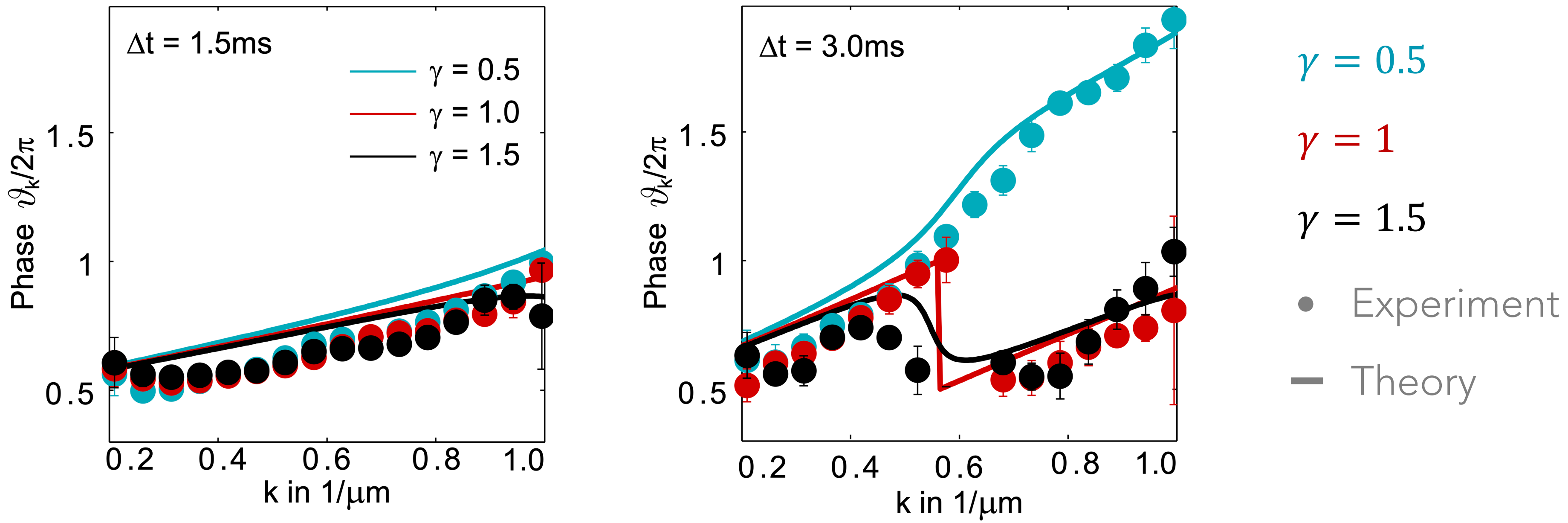


→ Dependence on initial state

→ Hard to distinguish between expansion scenarios

Phases

- For $\kappa < 0$ (harmonic trap)



→ Phases are robust and have decisive features

→ Analog cosmological particle production confirmed

Summary

Theory

- Mapping BEC $\leftrightarrow$ Universe

Phonons	$\leftrightarrow$	Relativistic scalar field
Density profile $n_0(r)$	$\leftrightarrow$	Spatial curvature κ
Coupling $\lambda(t)$	$\leftrightarrow$	Scale factor $a(t)$

- Cosmological particle production

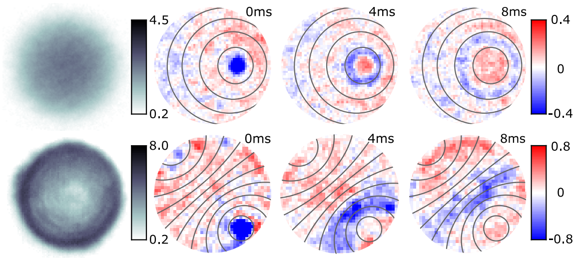
before — expansion $a(t)$ — after

$$\hat{a}_{km}|\Omega\rangle = 0 \xrightarrow[\alpha_k,\beta_k]{} \hat{b}_{km}|\Psi\rangle = 0$$

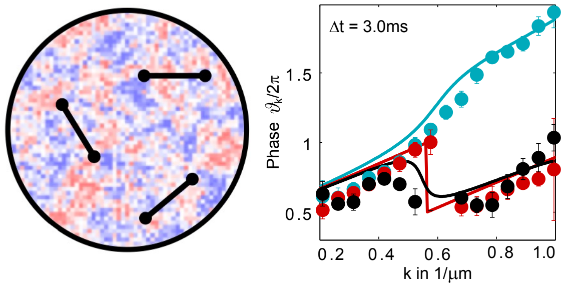
$$S_k(t) = \frac{1}{2} + |\beta_k|^2 + |\alpha_k\beta_k|\cos(2\omega_k t + \Theta_k)$$

Experiment

- Phonon propagation

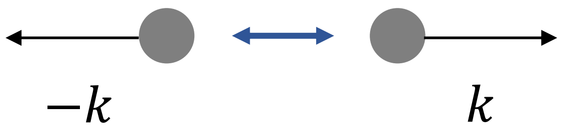


- Density fluctuations



Outlook

- Simulate other cosmologies
  - Vary spatial curvature κ in time
  - Expansions with accelerating ($\gamma > 1$) and decelerating ($\gamma < 1$) epochs
  - Horizons
- Certify quantum nature of produced particles
  - Particles are produced in pairs with opposite momenta



→ Detect entanglement

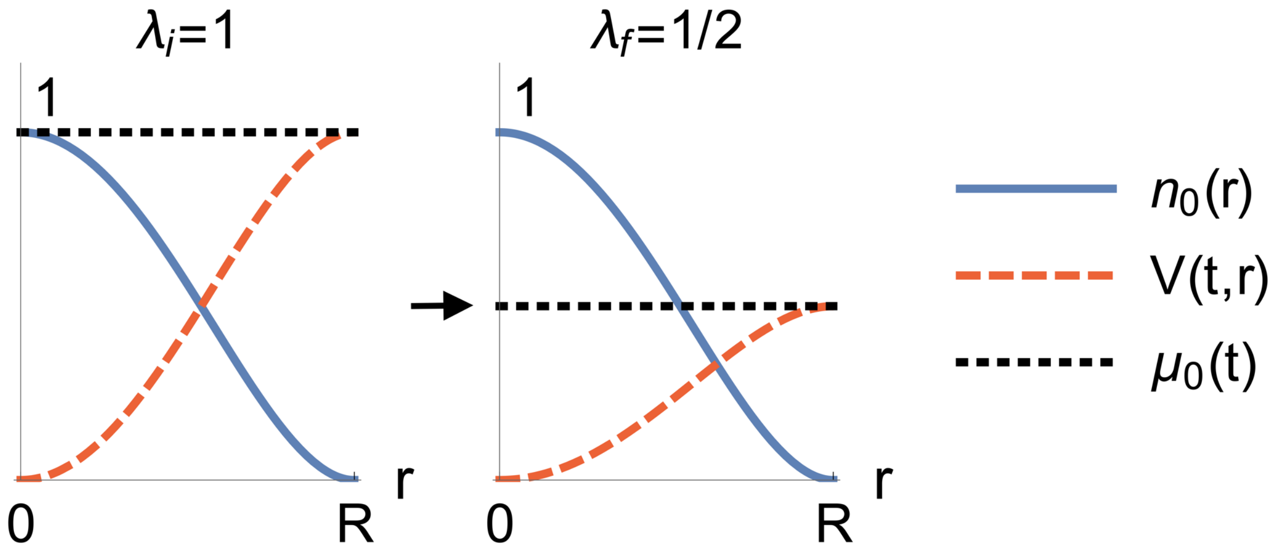
Thanks
for your attention

Backup

Keeping background density static

- Changing the coupling $\lambda(t)$ is accompanied by

$$\omega^2(t) = \frac{2\bar{n}_0}{mR^2}\,\lambda(t), \qquad \mu_0(t) = \bar{n}_0\,\lambda(t)$$



More details on the mapping

- Acoustic line element for appropriate density profiles $n_0(r)$

$$ds^2 = -dt^2 + \frac{m}{\bar{n}_0}\frac{1}{\lambda(t)}\left(1 \mp \frac{r^2}{R^2}\right)^{-2}(dr^2 + r^2 d\varphi^2)$$

- Define

$$a^2(t) = \frac{m}{\bar{n}_0}\frac{1}{\lambda(t)}, \qquad u(r) = \frac{r}{1 \mp \frac{r^2}{R^2}} \quad \rightarrow \quad \frac{dr^2}{\left(1 \mp \frac{r^2}{R^2}\right)^2} = \frac{du^2}{1 \pm 4\frac{u^2}{R^2}}$$

- Arrive at FLRW metric

$$ds^2 = -dt^2 + a^2(t)\left(\frac{du^2}{1-\kappa u^2} + u^2 d\varphi^2\right)$$

with $\kappa = \mp 4/R^2$

Experimental values

• Atom species	^{39}K
• Atom number	$\approx 23\,000$
• Trapping frequency in z-direction	$\omega_z = 2\pi \times 1.6$ kHz
• Trapping frequency in r-direction	$\omega_r = 2\pi \times (7 - 23)$ Hz
• Thomas-Fermi radius	$R_{TF} = (25, 30)\ \mu$m
• Scattering length	$a_s = (50 - 400)\ a_B$
• Imaging resolution	$1\ \mu$m