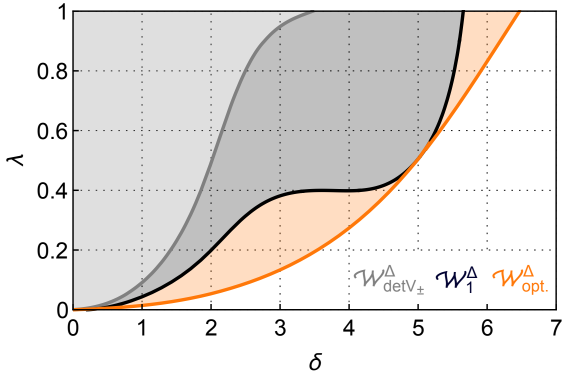
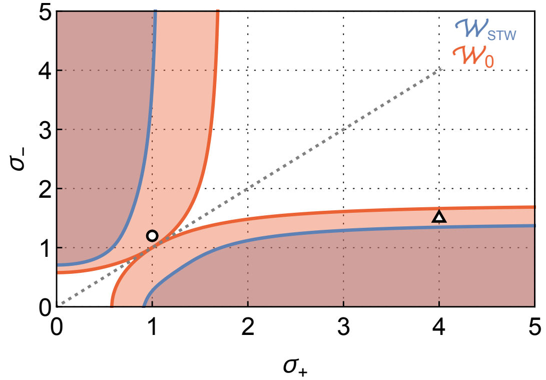
General class of continuous variable entanglement criteria

Martin Gärttner[1,2,3], Tobi Haas[4], Johannes Noll[2]

[1]ITP HD, [2]Kirchhoff-Institut HD, [3]Physikalisches Institut HD, [4]QuIC Bruxelles

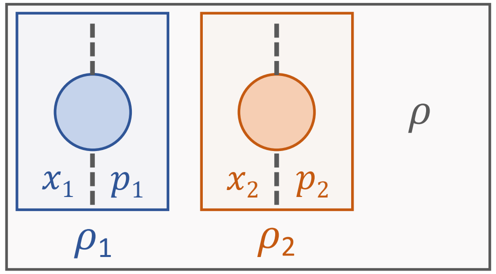
PRL 131, 150201, PRA 108, 042410





Big picture

- Bipartition of two bosonic modes
  - Conjugate variable pairs (x_1, p_1), (x_2, p_2)
  - Global state ϱ, local states $\varrho_1 = Tr_2\{\varrho\}$, $\varrho_2 = Tr_1\{\varrho\}$



- Global state ϱ separable if

$$\varrho = \sum_i p_i \left(\varrho_1^i \otimes \varrho_2^i\right)$$

- Certify entanglement
  - by efficiently accessing (x_1, p_1),(x_2, p_2) → measure Husimi Q
  - for as many states as possible → formulate general criteria
  - in situations with limited data → optimize post-measurement

Outline

Theory	Experimental perspective
• Husimi Q-distribution	• Finite detector resolution
• General criteria	• Finite statistics

Correlation ↔ Localization

- Local rotations ϑ_j

$$\begin{pmatrix} r_j \\ s_j \end{pmatrix} = \begin{pmatrix} \cos\vartheta_j & \sin\vartheta_j \\ -\sin\vartheta_j & \cos\vartheta_j \end{pmatrix} \begin{pmatrix} x_j \\ p_j \end{pmatrix}$$

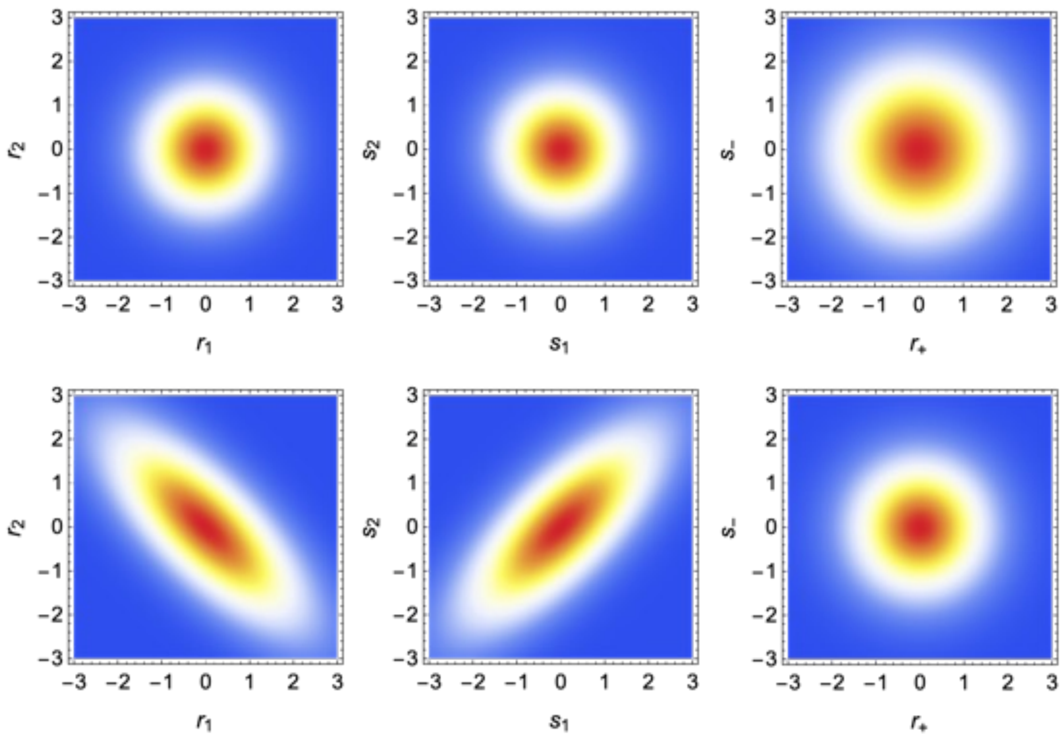
- Local rescalings a_1, a_2, b_1, b_2
- Non-local variables EPR ´35

$$r_\pm = a_1 r_1 \pm a_2 r_2$$

$$s_\pm = b_1 s_1 \pm b_2 s_2$$

- Ind. canonical pairs if $a_1 b_1 = a_2 b_2$

$$[R_\pm, S_\pm] = 2 i a_1 b_1,\ [R_\pm, S_\mp] = 0$$

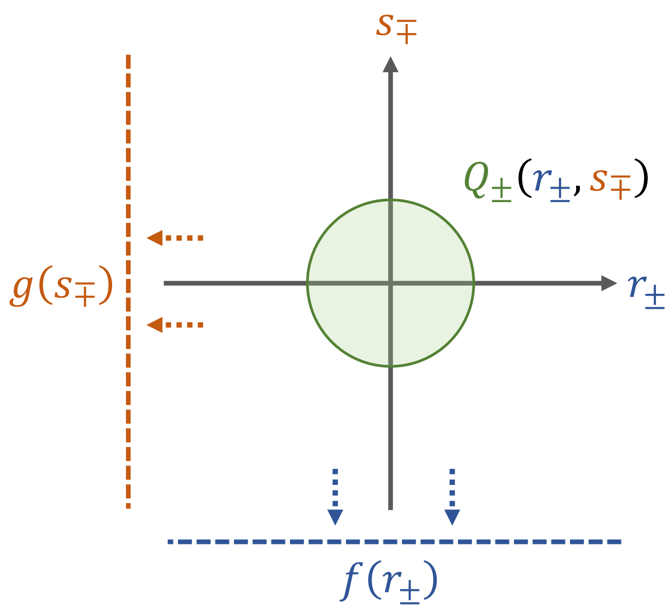


Homodyne vs. heterodyne

- State $\varrho \leftrightarrow$ Phase space representation Weedbrook ´12, Serafini ´17
- Homodyne $\rightarrow$ Marginals $f(r_\pm), g(s_\mp)$
- Heterodyne $\rightarrow$ Husimi $Q_\pm = Q_\pm(r_\pm, s_\mp)$
  - from $Q = (\langle\alpha_1|\otimes\langle\alpha_2|)\varrho(|\alpha_1\rangle\otimes|\alpha_2\rangle)$
  - coherent states $a_j|\alpha_j\rangle = \alpha_j|\alpha_j\rangle, \alpha_j = (x_j + ip_j)/\sqrt{2}$
  - and $Q_\pm = \int \frac{dr_\mp ds_\pm}{2\pi} Q(r_+, s_+, r_-, s_-)$

  $\rightarrow$ non-negative $Q_\pm(r_\pm, s_\mp) \geq 0$

  $\rightarrow$ normalized $\int \frac{dr_\pm ds_\mp}{2\pi} Q_\pm(r_\pm, s_\mp) = 1$



General criteria

- PPT Peres ´96, Horodecki ´96: ϱ separable $\Rightarrow \varrho^{T_2}$ physical
- In phase space: $Q(r_\pm, s_\pm) \to Q(r_\pm, s_\mp)$ physical
- Most general uncertainty principle: Lieb-Solovej theorem: Lieb, Solovej ´14
- Define witness functional for concave f with $f(0) = 0$

$$\mathcal{W}_f = \int \frac{dr_\pm ds_\mp}{2\pi} \left[f(Q_\pm) - f(\bar{Q}'_\pm)\right], \quad \text{with} \quad \bar{Q}'_\pm(r_\pm, s_\mp) = \frac{1}{a_1 b_1 + a_2 b_2} e^{-\frac{1}{2}\frac{r_\pm^2 + s_\mp^2}{a_1 b_1 + a_2 b_2}}$$

- General criteria

$$\varrho \text{ separable} \Rightarrow \mathcal{W}_f \geq 0 \text{ for all } f$$

- Continuous majorization theory: ϱ separable $\Rightarrow Q_\pm \prec \bar{Q}'_\pm$

Implied criteria

- Monomials $f(t) = t^\beta$ and monotonic function → Rényi-Wehrl criteria Wehrl ´78, ´79

$$\mathcal{W}_\beta = S_\beta(Q_\pm) - \frac{\ln\beta}{\beta-1} - \frac{\ln\det\bar{V}'_\pm}{2} \geq 0, \quad \text{with} \quad S_\beta(Q_\pm) = \frac{1}{1-\beta}\ln\left[\int \frac{dr_\pm ds_\mp}{2\pi} Q_\pm^\beta(r_\pm, s_\mp)\right]$$

- For given covariance matrix

$$V_\pm = \begin{pmatrix} \sigma_{r_\pm}^2 + \frac{a_1^2+a_2^2}{2} & \sigma_{r_\pm s_\mp} \\ \sigma_{r_\pm s_\mp} & \sigma_{s_\mp}^2 + \frac{b_1^2+b_2^2}{2} \end{pmatrix},$$

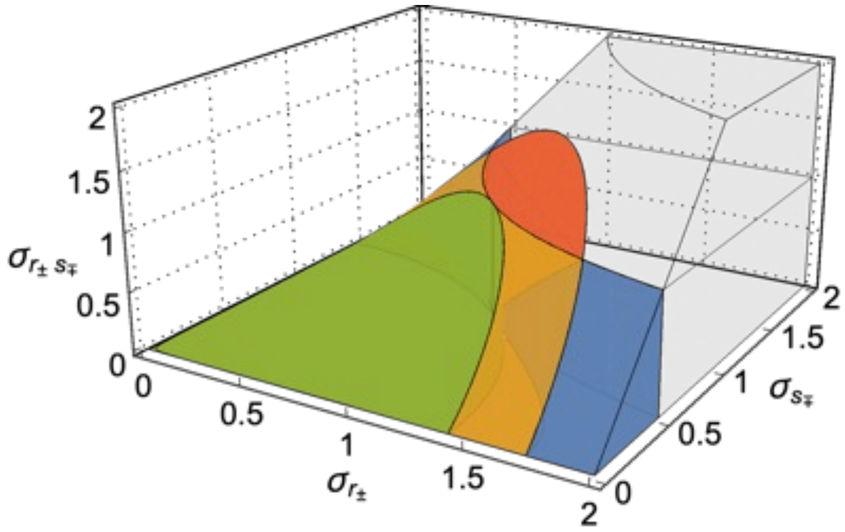
Wehrl entropy maximized for Gaussians → Second moment criteria

$$\mathcal{W}_{\det V_\pm} = \det V_\pm - \det\bar{V}'_\pm \geq 0, \quad \text{where} \quad \bar{V}'_\pm = (a_1 b_1 + a_2 b_2)\mathrm{I}$$

- For Gaussian states: necessary and sufficient + independent of f

Comparison I: Second moment criteria

- Implies DGCZ[Duan et al. ´00], equivalent to MGVT[Mancini et al. ´02] after optimization



witnessed by

→ Outperformance for large covariance $\sigma_{r_\pm s_\mp}$

Comparison II: Entropic criteria

- Rényi entropic criteria by Saboia, Toscano and Walborn (STW)Saboia et al. ´11

$$\mathcal{W}_{STW} = S_\alpha(f_\pm) + S_\beta(g_\mp) - \frac{1}{2(1-\alpha)} \ln\frac{\alpha}{\pi} - \frac{1}{2(1-\beta)} \ln\frac{\beta}{\pi} - \frac{\ln\det \bar{V}'_\pm}{2} \quad \text{for } \frac{1}{\alpha} + \frac{1}{\beta} = 2.$$

- Special case: Entropic critera by Walborn, Taketani, Salles, Toscano and de Matos Filho (WTSTD)Walborn et al. ´09

$$\mathcal{W}_{WTSTD} = S(f_\pm) + S(g_\mp) - 1 - \ln\pi - \frac{\ln\det \bar{V}'_\pm}{2}$$

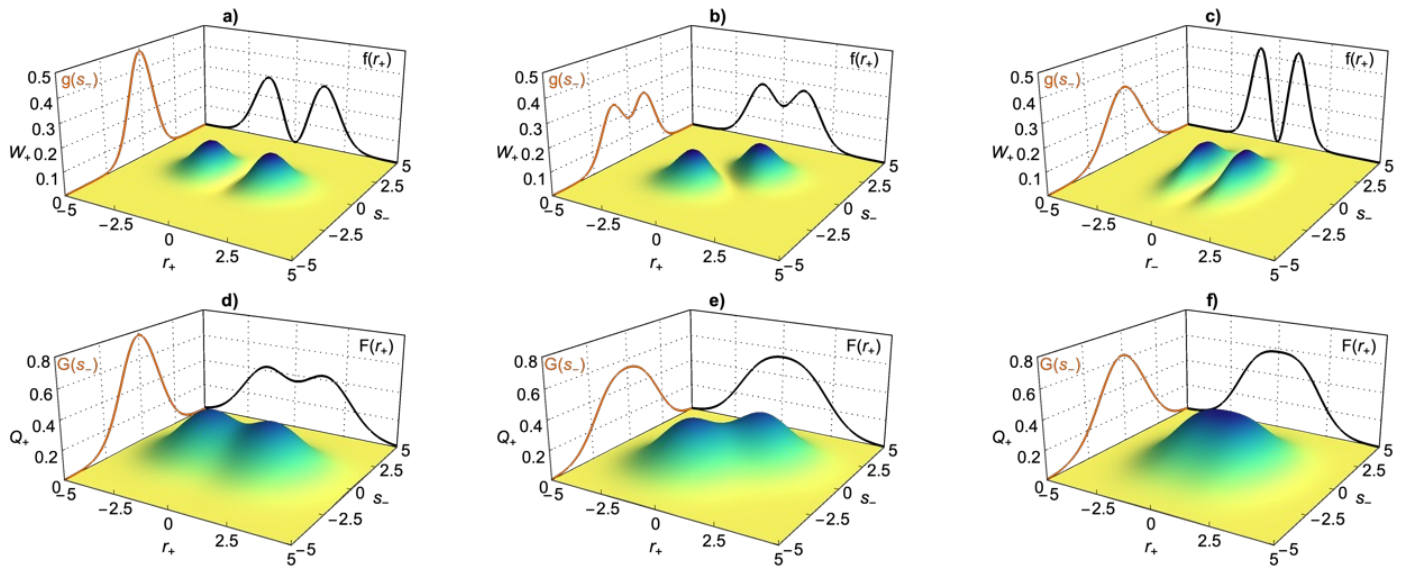
- We find the relation

$$\mathcal{W}_1 \geq \frac{1}{2}\mathcal{W}_{WTSTD} - I(F_\pm : G_\mp)$$

→ Outperformance expected for large correlations in the $\pm$ variables

Example state - I

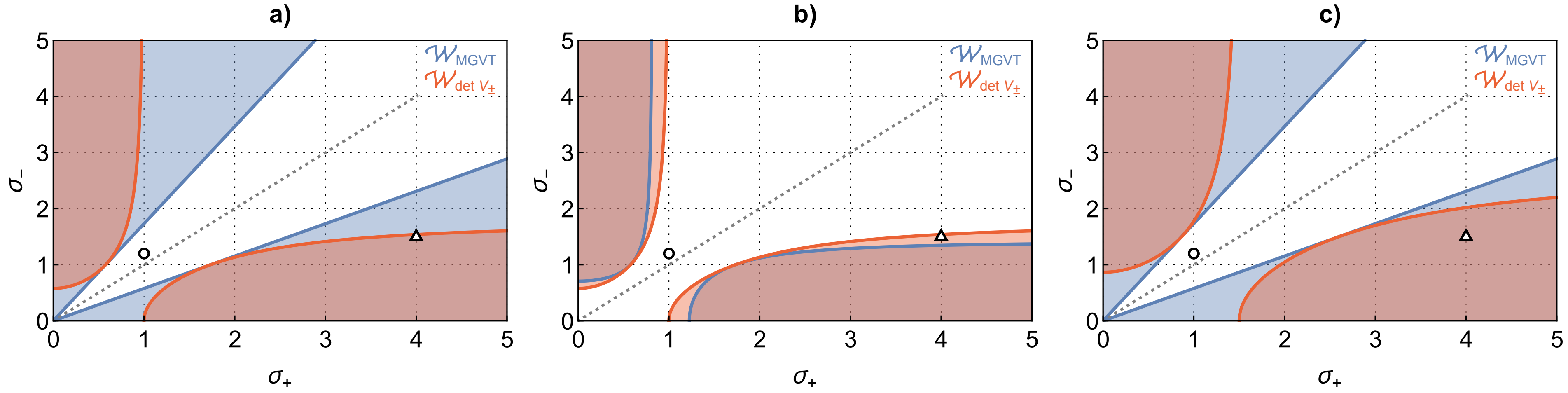
- Non-Gaussian state: $\psi(r_1, r_2) = \frac{r_1 + r_2}{\sqrt{\pi \sigma_- \sigma_+^3}} e^{-\frac{1}{4}\left[\left(\frac{r_1+r_2}{\sigma_+}\right)^2 + \left(\frac{r_1-r_2}{\sigma_-}\right)^2\right]}$
- Generalize to arbitrary angle ϕ and squeezing ξ



$\phi = 0, \xi = 1$ $\qquad$ $\phi = \frac{\pi}{4}, \xi = 1$ $\qquad$ $\phi = 0, \xi = \frac{3}{2}$

Example state - II

- Second moment criteria



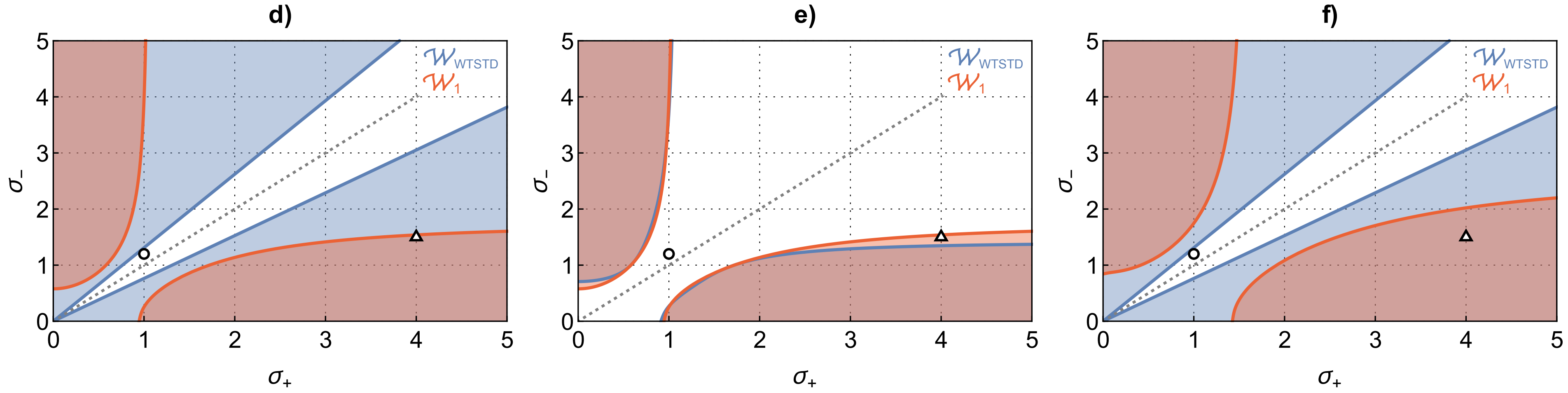
$\phi = 0, \xi = 1$

$\phi = \frac{\pi}{4}, \xi = 1$

$\phi = 0, \xi = \frac{3}{2}$

Example state - III

- Entropic criteria



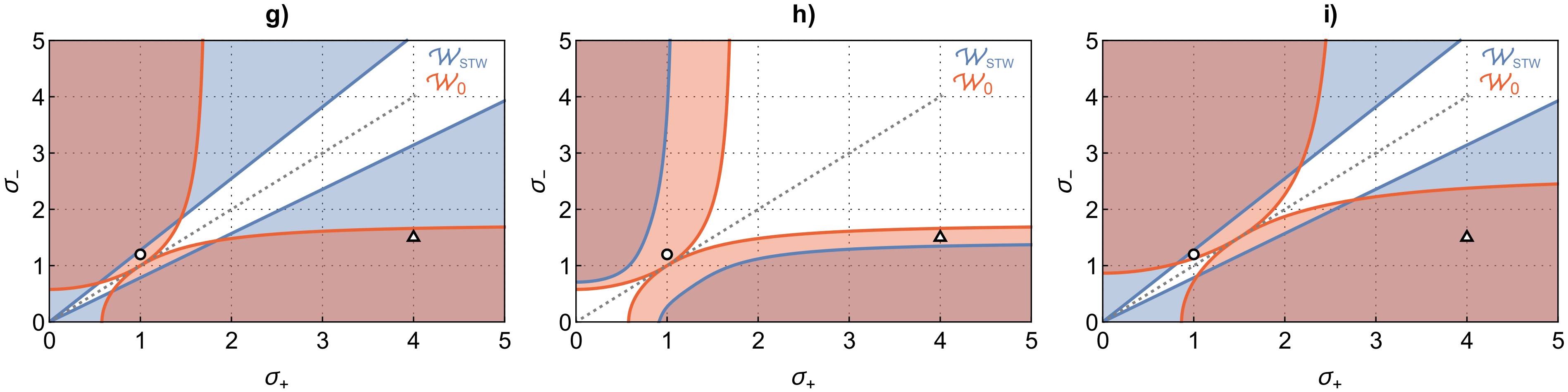
$\phi = 0, \xi = 1$

$\phi = \frac{\pi}{4}, \xi = 1$

$\phi = 0, \xi = \frac{3}{2}$

Example state - IV

- Rényi-entropic criteria

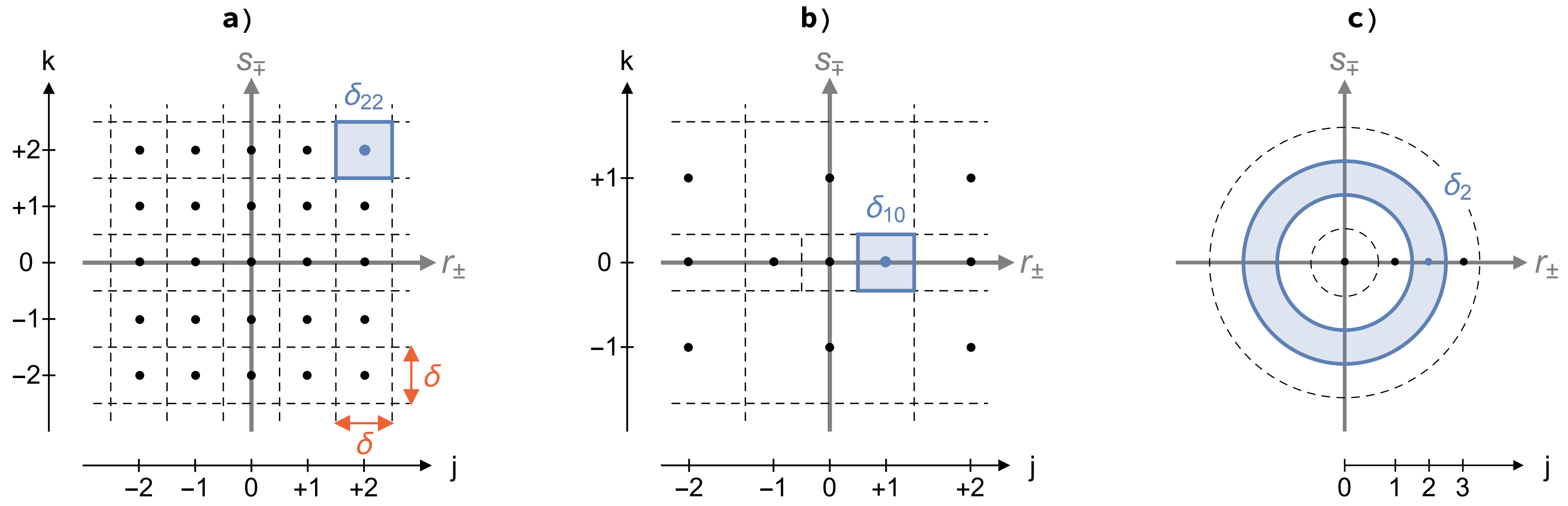


$\phi = 0, \xi = 1$ $\qquad$ $\phi = \frac{\pi}{4}, \xi = 1$ $\qquad$ $\phi = 0, \xi = \frac{3}{2}$

→ Strong outperformance: state witnessed for all $\sigma_+ \neq \sigma_-$ after optimization

Discretization schemes

- Discretize phase space into *arbitrary* compact regions δ_{jk}
- For example:



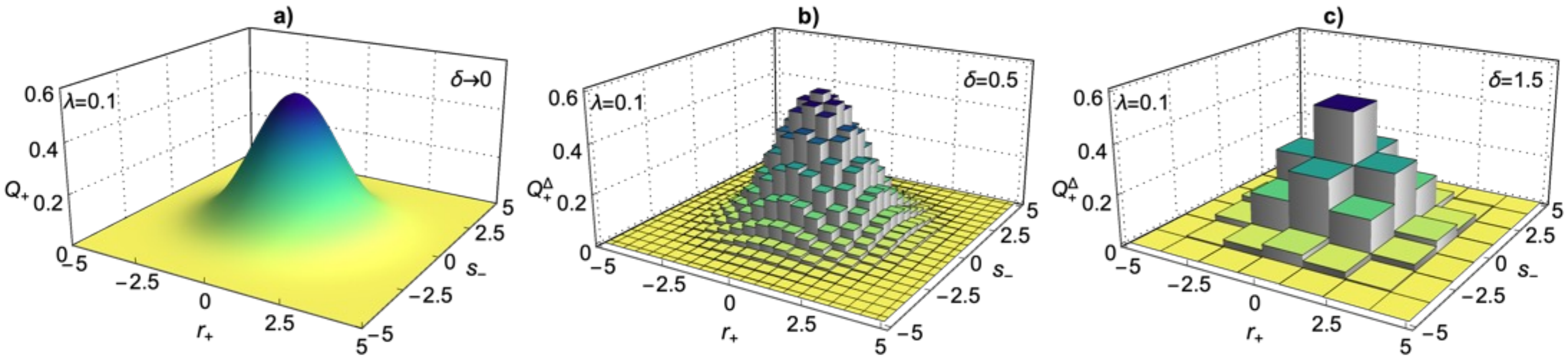
quadratic adaptive concentric

Discretized distributions

- We do *not* consider discrete distributions Tasca et al. ´13 $Q_{\pm}^{jk} = \int_{\delta_{jk}} \frac{dr_{\pm} ds_{\mp}}{2\pi} Q_{\pm}(r_{\pm}, s_{\mp})$

  but instead their densities $Q_{\pm}^{\Delta}(r_{\pm}, s_{\mp}) = \sum_{j,k} \begin{cases} \frac{Q_{\pm}^{jk}}{\Delta_{jk}} & \text{for } (r_{\pm}, s_{\mp}) \in \delta_{jk} \\ 0 & \text{else} \end{cases}$

  → Give discrete approximations to $Q_{\pm}(r_{\pm}, s_{\mp})$



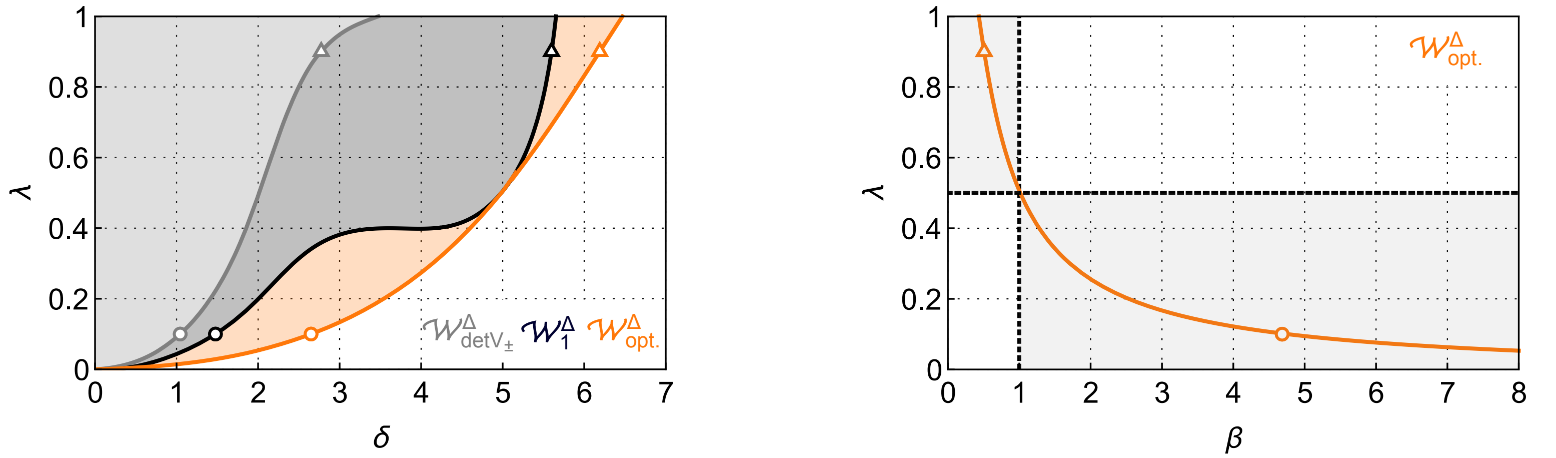
Discretized criteria

- Discretized criteria w.r.t. $Q^{\Delta}_{\pm}$ from Jensen's inequality

$$\varrho \text{ separable} \Rightarrow \mathcal{W}^{\Delta}_{f} \geq 0 \text{ for all } f$$

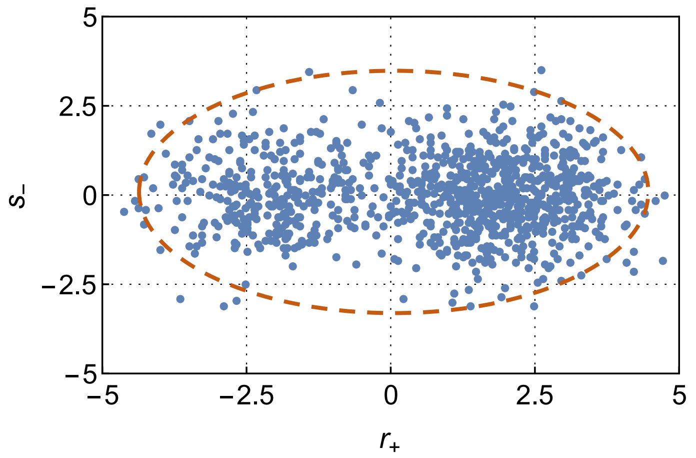
→ Optimize over f for given Δ

- Example: TMSV

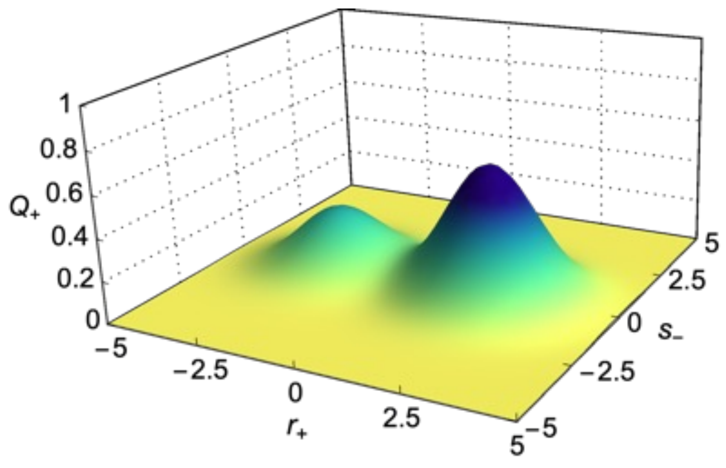


Sampled distributions

- Estimate $Q_\pm$ from samples (e.g. with machine learning algorithms)



→



- Main problem: Tails of $Q_\pm$ are hard to estimate from sparse data

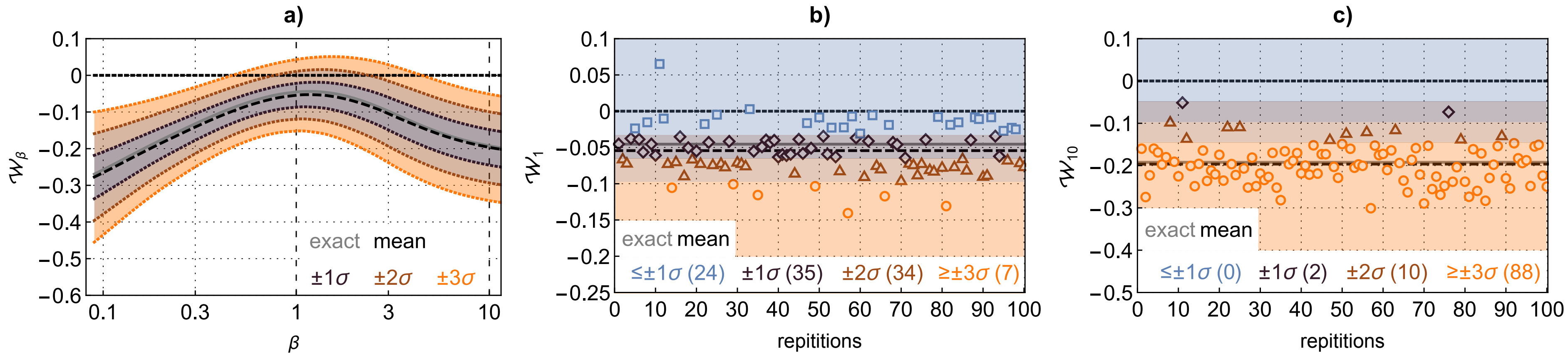
  → Choose f which suppresses the tails, e.g. large $\beta \gg 1$

Example state

- Displaced mixture of TMSV states with equal squeezing λ

$$Q_{\pm}(r_{\pm}, s_{\mp}) = (1-p)\frac{1+\lambda}{2}e^{-\frac{1+\lambda}{4}\left[(r_{\pm}-r)^2+s_{\mp}^2\right]} + p\frac{1+\lambda}{2}e^{-\frac{1+\lambda}{4}\left[(r_{\pm}+r)^2+s_{\mp}^2\right]}$$

- Results for Rényi-Wehrl witness (1000 samples, 100 repititions)

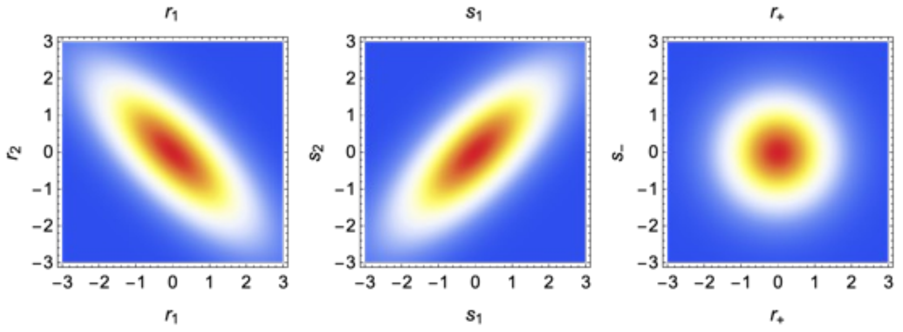


→ Signal-to-noise ratio improved for large β

Summary

Theory

- Husimi Q-distribution



- General criteria

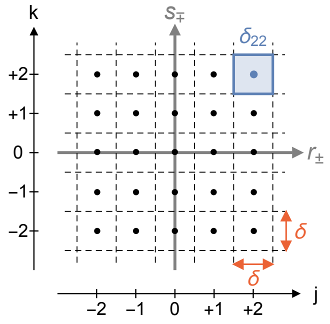
$$\mathcal{W}_f = \int \frac{dr_\pm ds_\mp}{2\pi} \left[f(Q_\pm) - f(\bar{Q}'_\pm) \right] \geq 0$$

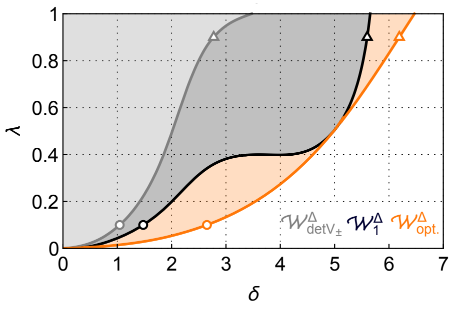
→ Entropic & 2nd moment criteria

→ Optimize over f

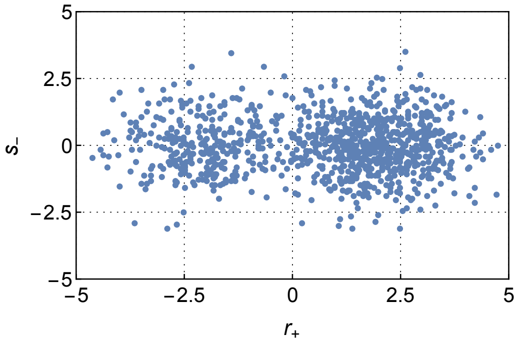
Experimental perspective

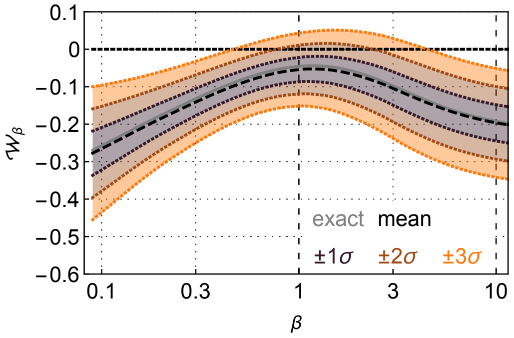
- Finite detector resolution





- Finite statistics

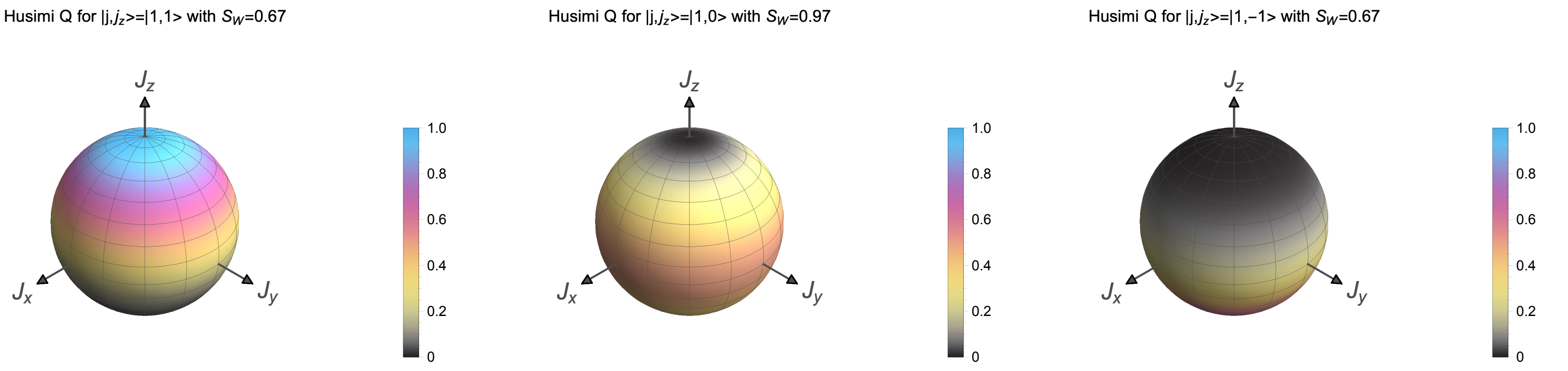




Outlook

- Phase space descriptions and Lieb-Solovej theorems exist for almost all simple Lie groups[Zhang ´90]

  → Generalize our approach to other systems, e.g. quantum spins



- Apply phase space tools to quantum field theoretic problems

  → EURs and witnesses for quantum fields[Floerchinger ´22, TH work in progress]

Thanks
for your
attention :)